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ing one, and in the not very frequent instances in which a dark marking is seen to extend actually *to* the limb itself, it is invariably the following one. This would seem to point to the fact that sunrise on *Mars* is much clearer than sunset; but I abstain from any theory as to the cause of this.

*Forest Lodge, Maresfield, Uckfield,*
*November* 8, 1877.

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## *On the Motion of the Moon's Node in the case when the Orbits of the Sun and Moon are supposed to have no Eccentricities, and when their mutual Inclination is supposed to be indefinitely small.* By Professor J. C. Adams, M.A.

A very able paper has recently been published by Mr. G. W. Hill, assistant in the office of the *American Nautical Almanac*, on the part of the motion of the lunar perigee which is a function of the mean motions of the Sun and Moon.

Assuming that the values of the Moon's coordinates in the case of no eccentricities are already known, the author finds the differential equations which determine the inequalities which involve the first power of the eccentricity of the Moon's orbit, and, by a most ingenious and skilful process, he makes the solution of those differential equations depend on the solution of a single linear differential equation of the second order, which is of a very simple form. This equation is equivalent to an infinite number of algebraical linear equations, and the author, by a most elegant method, shows how to develop the infinite determinant corresponding to these equations in a series of powers and products of the small quantities forming their coefficients. The value of the multiplier of each of such powers and products as are required is obtained in a finite form. By equating this determinant to zero, an equation is obtained which gives directly, and without the need of successive approximations, the motion of the Moon from the perigee during half of a synodic month. The small quantities which enter into the value of the above determinant are of the fourth, eighth, twelfth, &c. orders, considering, as usual, the ratio of the mean motion of the Sun to that of the Moon as a small quantity of the first order; and the author has taken into account all the terms of lower orders than the sixteenth. The ratio of the motion of the perigee to that of the Moon thus obtained is true to twelve or thirteen significant figures. The author compares his numerical result with that deduced from Delaunay's analytical formula, which gives the ratio just mentioned developed in a series of powers of $m$, the ratio of the mean motions of the Sun and Moon. The numerical

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coefficients of the successive terms of this series increase so rapidly that the convergence of the series is slow, so that the terms calculated do not suffice to give the first four significant figures of the result correctly, although, by induction, a rough approximation may be made to the sum of the remaining terms of the series.

I have been led to dwell thus particularly on Mr. Hill's investigation because my own researches in the Lunar Theory have followed, in some respects, a parallel course, *sed longo intervallo.*

I have long been convinced that the most advantageous way of treating the Lunar Theory is, first, to determine with all desirable accuracy the inequalities which are independent of the eccentricities $e$ and $e'$, and the inclination $2\sin^{-1}\gamma$, and then, in succession, to find the inequalities which are of one dimension, two dimensions, and so on, with respect to those quantities.

Thus the coefficient of any inequality in the Moon's coordinates would be represented by a series arranged in powers and products of $e$, $e'$, and $\gamma$, and each term in this series would involve a numerical coefficient which is a function of $m$ alone and which may be calculated for any given value of $m$ without the necessity of developing it in powers of $m$. The variations of these coefficients which would result from a very small change in $m$ might be found either independently or by making the calculation for two values of $m$ differing by a small quantity.

This method is particularly advantageous when we wish to compare our results with those of an analytical theory such as Delaunay's, in which the eccentricities and the inclination are left indeterminate, since each numerical coefficient so obtained could be compared separately with its analytical development in powers of $m$.

It is to be remarked that it is only the series proceeding by powers of $m$ in Delaunay's Theory which have a slow rate of convergence, so that it is probable that all the sensible corrections required by Delaunay's coefficients would be found among the terms of low order in $e$, $e'$, and $\gamma$.

The differential equations which would require solution in these successive operations after the determination of the inequalities independent of eccentricities and inclination would be all linear and of the same form.

It is many years since I obtained the values of these last-named inequalities to a great degree of approximation, the coefficients of the longitude expressed in circular measure, and those of the reciprocal of the radius vector, or of the logarithm of the radius vector, being found to ten or eleven places of decimals.

In the next place I proceeded to consider the inequalities of latitude, or rather the disturbed value of the Moon's coordinate perpendicular to the ecliptic, omitting the eccentricities as before, and taking account only of the first power of $\gamma$.

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In this case the differential equation for finding $z$ presents itself naturally in the form to which Mr. Hill reduces, with so much skill, the equations depending on the first power of the eccentricity of the Moon's orbit.

In solving this equation I fell upon the same infinite determinant as that considered by Mr. Hill, and I developed it in a similar manner in a series of powers and products of small quantities, the coefficient of each such term being given in a finite form.

The terms of the fourth order in the determinant were thus obtained by me on the 26th December 1868. I then laid aside the further investigation of this subject for a considerable time, but resumed it in 1874 and 1875, and on the 2nd of December in the latter year I carried the approximation to the value of the determinant as far as terms of the twelfth order, or to the same extent as that which has been attained by Mr. Hill. I have also succeeded in reducing the determination of the inequalities of longitude and radius vector which involve the first power of the lunar eccentricity to the solution of a differential equation of the second order, but my method is much less elegant than that of Mr. Hill.

Immediately after Mr. Hill's paper reached me, I wrote to him expressing my opinion of its merits, and telling him what I had done in the same direction, and I received from him a very cordial and friendly letter in reply.

The equation which I had obtained by equating the above-mentioned determinant to zero differed in form from Mr. Hill's, and on making the reductions required to make the two results immediately comparable, I found that there was an agreement between them except in one term of the twelfth order. On examining my work I found that this arose from a simple error of transcription in a portion of my work, and that when this had been rectified my result was in entire accordance with Mr. Hill's.

The calculations by which I have found the value of the determinant are very different in detail from those required by Mr. Hill's method, and appear to be considerably more laborious. I have not yet had time to copy out and arrange the details of the calculations from my old papers, but I hope soon to do so, thinking that they may not be without interest for the Society. Meantime I now make known the result which I have obtained for the motion of the Moon's node on the suppositions stated in the title of this paper.

If $nt$ and $n't$ represent the mean longitudes of the Moon and the Sun at time $t$, omitting, for the sake of brevity in writing, the constants which always accompany $nt$ and $n't$, and if $\theta$ and $r$ represent the Moon's longitude and radius vector, I find that, in the case of no eccentricities and inclination, if $m = \frac{n'}{n} = 0{\cdot}0748013$, which is the value used by Plana,

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$$
\begin{aligned}
\theta = nt &+ 0{\cdot}01021{,}13629{,}5 \sin 2(n-n')\,t \\
&+ 0{\cdot}00004{,}23732{,}7 \sin 4(n-n')\,t \\
&+ 0{\cdot}00000{,}02375{,}7 \sin 6(n-n')\,t \\
&+ 0{\cdot}00000{,}00015{,}1 \sin 8(n-n')\,t \\
&+ 0{\cdot}00000{,}00000{,}1 \sin 10(n-n')\,t\,;
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{r} = \quad & 1{\cdot}00090{,}73880{,}5 \\
&+ 0{\cdot}00718{,}64751{,}6 \cos 2(n-n')\,t \\
&+ 0{\cdot}00004{,}58428{,}9 \cos 4(n-n')\,t \\
&+ 0{\cdot}00000{,}03268{,}6 \cos 6(n-n')\,t \\
&+ 0{\cdot}00000{,}00024{,}3 \cos 8(n-n')\,t \\
&- 0{\cdot}00000{,}00000{,}3 \cos 10(n-n')\,t\,;
\end{aligned}
$$

supposing that $\theta$ is expressed in the circular measure, and that the unit of distance is the mean distance in an undisturbed orbit which would be described by the Moon about the Earth in the same periodic time. In this case, if $\mu$ denote the sum of the masses of the Earth and Moon, we shall have

$$\mu = n^2.$$

The differential equation which determines $z$, the Moon's coordinate perpendicular to the Ecliptic, is

$$\frac{d^2z}{dt^2} + \left(\frac{\mu}{r^3} + \frac{m'}{r_1^{\,3}}\right) z = 0.$$

Now, the Sun's orbit being circular, we have $\dfrac{m'}{r_1^{\,3}} = n'^2$, and the only function of the Moon's coordinates which we require in order to form this equation is $\dfrac{1}{r^3}$.

I find that, with the above unit of distance,

$$
\begin{aligned}
\frac{1}{r^3} = \quad & 1{\cdot}00280{,}21783{,}115 \\
&+ 0{\cdot}02159{,}98364{,}4 \cos 2(n-n')\,t \\
&+ 0{\cdot}00021{,}53273{,}9 \cos 4(n-n')\,t \\
&+ 0{\cdot}00000{,}20644{,}8 \cos 6(n-n')\,t \\
&+ 0{\cdot}00000{,}00192{,}9 \cos 8(n-n')\,t \\
&+ 0{\cdot}00000{,}00000{,}3 \cos 10(n-n')\,t.
\end{aligned}
$$

Let

$$\frac{1}{(n-n')^2}\left(\frac{\mu}{r^3} + \frac{m'}{r_1^{\,3}}\right), \text{ or } \frac{1}{(n-n')^2}\left(\frac{n^2}{r^3} + n'^2\right), = \frac{1}{(1-m)^2}\left(\frac{1}{r^3} + m^2\right),$$

$$= q^2 + 2q_1 \cos 2(n-n')\,t + 2q_2 \cos 4(n-n')\,t + 2q_3 \cos 6(n-n')\,t + \&c.\,;$$

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then we find, from the above value of $\frac{1}{r^3}$, that

$$q^2 = 1{\cdot}17804{,}44973{,}149, \quad \text{and} \quad q = 1{\cdot}08537{,}75828{,}323,$$
$$q_1 = 0{\cdot}01261{,}68354{,}6,$$
$$q_2 = 0{\cdot}00012{,}57764{,}3,$$
$$q_3 = 0{\cdot}00000{,}12059{,}0.$$

These are all the quantities necessary for finding the motion of the Moon's node, to the order which we require.

If $g\pi$ denote the angular motion of the Moon from its node in half a synodic period of the Moon, the equation so often referred to above gives

$$\begin{aligned}\cos g\pi = \cos q\pi \Big\{ & 1 - \frac{\pi^2 q_1^4}{32q^2(q^2-1)^2} - \frac{15q^4-35q^2+8}{256q^4(q^2-1)^4(q^2-4)}\pi^2 q_1^6 \\ & + \frac{3\pi^2 q_1^4 q_2}{32q^2(q^2-1)^2(q^2-4)} - \frac{\pi^2 q_1^2 q_2^2}{16q^2(q^2-1)(q^2-4)} \Big\} \\ + \sin q\pi \Big\{ & \frac{\pi q_1^2}{4q(q^2-1)} + \frac{15q^4-35q^2+8}{64q^3(q^2-1)^3(q^2-4)}\pi q_1^4 - \frac{\pi^3 q_1^6}{384q^3(q^2-1)^3} \\ & + \frac{105q^{10}-1155q^8+3815q^6-4705q^4+1652q^2-288}{256q^5(q^2-1)^5(q^2-4)^2(q^2-9)}\pi q_1^6 \\ & - \frac{3\pi q_1^2 q_2}{8q(q^2-1)(q^2-4)} - \frac{35q^6-280q^4+497q^2-108}{32q^3(q^2-1)^3(q^2-4)^2(q^2-9)}\pi q_1^4 q_2 \\ & + \frac{\pi q_2^2}{4q(q^2-4)} + \frac{15q^6-110q^4+179q^2-36}{16q^3(q^2-1)^2(q^2-4)^2(q^2-9)}\pi q_1^2 q_2^2 \\ & + \frac{\pi q_3^2}{4q(q^2-9)} - \frac{(3q^2-7)\pi q_1 q_2 q_3}{4q(q^2-1)(q^2-4)(q^2-9)} + \frac{5\pi q_1^3 q_3}{16q(q^2-1)(q^2-4)(q^2-9)} \Big\}\end{aligned}$$

Now, if the coefficients of $\cos q\pi$ and $\sin q\pi$ in this formula be converted into numbers, employing the above values of $q$, $q_1$, &c., we find

$$\begin{aligned}\cos g\pi = {} & \cos q\pi \, [0{\cdot}99999{,}97902{,}01654] \\ & + \sin q\pi \, [0{\cdot}00064{,}77652{,}06681].\end{aligned}$$

But, with the above value of $q$, we find, from Briggs' Tables,

$$\cos q\pi = -0{\cdot}96424{,}37306{,}84295$$
$$\sin q\pi = -0{\cdot}26501{,}70331{,}05484.$$

Hence

$$\cos g\pi = -0{\cdot}96441{,}51972{,}00779.$$

Whence, by the same Tables, we find that

$$g = 1{\cdot}08517{,}13927{,}46869,$$

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and therefore the ratio of the Moon's motion from the node to its sidereal motion is

$$g(1-m) = 1{\cdot}00399{,}91618{,}46592.$$

This is the quantity ordinarily denoted by $g$ in the Lunar Theory.

Delaunay's value of $g$, which agrees with that of Plana, is

$$g = 1 + \frac{3}{4}m^2 - \frac{9}{32}m^3 - \frac{273}{128}m^4 - \frac{9797}{2048}m^5 - \frac{199273}{24576}m^6 - \frac{6657733}{589824}m^7.$$

If this be converted into numbers by substituting the value of $m = 0{\cdot}0748013$, we find

$$g = 1{\cdot}00399{,}91722{,}8,$$

which differs from the true value in the eighth place of decimals.

If we take $\mathrm{m} = \dfrac{m}{1-m}$ and develop the value of $g$ in powers of $\mathrm{m}$, we find

$$g = 1 + \frac{3}{4}\mathrm{m}^2 - \frac{57}{32}\mathrm{m}^3 + \frac{123}{128}\mathrm{m}^4 - \frac{1925}{2048}\mathrm{m}^5 + \frac{25667}{24576}\mathrm{m}^6 - \frac{268309}{589824}\mathrm{m}^7;$$

and substituting the value of

$$\mathrm{m} = 0{\cdot}08084{,}89030{,}52,$$

we find

$$g = 1{\cdot}00399{,}91591{,}1,$$

which is considerably nearer the truth than the value found from the series in powers of $m$.

The numerical values of the successive terms of the series for $g - 1$, in terms of powers of $m$ and of $\mathrm{m}$ respectively, are given in the following comparative table :—

| In powers of $m$. | | | In powers of $\mathrm{m}$. | | |
|---|---|---|---|---|---|
| $m^2$ | | ·00419,64258,6 | $\mathrm{m}^2$ | | ·00490,24088,4 |
| $m^3$ | − | 11,77117,9 | $\mathrm{m}^3$ | − | 94,13416,4 |
| $m^4$ | − | 6,67712,1 | $\mathrm{m}^4$ | | 4,10574,2 |
| $m^5$ | − | 1,12023,4 | $\mathrm{m}^5$ | − | ,32469,2 |
| $m^6$ | − | ,14203,4 | $\mathrm{m}^6$ | | ,02916,8 |
| $m^7$ | − | ,01479,0 | $\mathrm{m}^7$ | − | ,00102,7 |
| | | ·00399,91722,8 | | | ·00399,91591,1 |

This shows that the development in powers of $\mathrm{m}$ is much more advantageous than that in powers of $m$.

The same thing likewise holds good with respect to the value of $c$, which determines the motion of the perigee.

The following is a similar table, showing the numerical values

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of the successive terms of Delaunay's series for $1-c$ in powers of $m$, and of the terms of the corresponding series in powers of $\mathrm{m}$:—

| In powers of $m$. | | In powers of $\mathrm{m}$. | |
|---|---|---|---|
| $m^2$ | ·00419,64258,6 | $\mathrm{m}^2$ | ·00490,24088 |
| $m^3$ | 294,27947,8 | $\mathrm{m}^3$ | 292,31135 |
| $m^4$ | 99,56981,8 | $\mathrm{m}^4$ | 55,37745 |
| $m^5$ | 30,35769,9 | $\mathrm{m}^5$ | 14,37162 |
| $m^6$ | 9,13946,6 | $\mathrm{m}^6$ | 3,49278 |
| $m^7$ | 2,82999,6 | $\mathrm{m}^7$ | ,99062 |
| $m^8$ | ,98356,5 | $\mathrm{m}^8$ | ,42111 |
| $m^9$ | ,34684,2 | $\mathrm{m}^9$ | ,08515 |
| | ·00857,14945,0 | | ·00857,29096 |

The true value reduced from Mr. Hill's, so as to correspond to the value of $m$ which we have employed, is

·00857,25645.

Hence, as in the former case, the advantage of developing in powers of m is very evident.

I have found that a similar advantage results from the employment of m instead of $m$ in the development of the co-efficients of the Moon's periodic inequalities.

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## *On a New Term of Long Period in the Mean Motion of the Moon.*
### By E. Neison, Esq.

The mean motion of the Moon contains a term due to the direct action of the planet *Mars*, and possessing the argument

$$\{(n-24n''+20n''')\,t+\mathrm{E}\},$$

where, following Leverrier's nomenclature, two accents denote quantities belonging to the Earth, and three accents those belonging to the planet *Mars*. The following note is directed to determining provisionally the value of this term:—

Let $\mathrm{R}^{(''')}$ denote the disturbing function arising from the direct action of *Mars* upon the Moon. Then, as shown by Delaunay,

$$\mathrm{R}^{(''')}=m'''\left\{\frac{r^2}{\Delta^3}\left(\frac{3}{2}\cos^2\mathrm{H}-\frac{1}{2}\right)+\frac{r^3}{\Delta^4}\left(\frac{5}{2}\cos^3\mathrm{H}-\frac{3}{2}\cos\mathrm{H}\right)\right\},$$

where

$\Delta$ = geocentric distance of *Mars*,

H = geocentric angular distance between *Mars* and the Moon.

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Suppose the inclinations of the Moon and planet to be neglected, which can be done without sensibly affecting the results, then

$$\cos \mathrm{H} = \frac{r'''}{\Delta} \cos (v - v''') - \frac{r''}{\Delta} \cos (v - v''),$$

whence

$$\cos^3 \mathrm{H} = \frac{1}{\Delta^3} \left\{ \left( \frac{3}{4} r'''^3 + \frac{3}{2} r''^2 r''' \right) \cos (v - v''') - \left( \frac{3}{4} r''^3 + \frac{3}{2} r'''^2 r'' \right) \cos (v - v'') \right.$$
$$\left. - \frac{3}{4} r'''^2 r'' \cos (v - 2v''' + v'') + \frac{3}{4} r''^2 r''' \cos (v - 2v'' + v''') \right\} \cdot$$

Therefore, by substitution

$$\mathrm{R}^{(''')} = m''' r^3 \left\{ \left( \frac{15}{8} \cdot \frac{r'''^3 + 2r''' r''^2}{\Delta^7} - \frac{3}{2} \frac{r'''}{\Delta^5} \right) \cos (v - v''') - \left( \frac{15}{8} \frac{r''^3 + 2r'' r'''^2}{\Delta^7} - \frac{3}{2} \frac{r''}{\Delta^5} \right) \cos (v - v'') \right.$$
$$\left. + \frac{15}{8} \frac{r'' r'''}{\Delta^7} \left[ r'' \cos (v - 2v'' + v''') - r''' \cos (v - 2v''' + v'') \right] + \ldots \right.$$

Throughout only terms depending on $v$ have been retained.

As $\Delta$ is the geocentric distance of *Mars*,

$$\Delta^{-s} = \{ r'''^2 + r''^2 - 2r'' r''' \cos (v'' - v''') \}^{-\frac{s}{2}},$$

a well-known function.

Let

$$(a''')^{-s} \, [\mathrm{B}_{\frac{s}{2}}]^{i}_{i+k} \cos \{ [in'' - (i+k) n'''] \, t + \mathrm{E} \}$$

be the general term of $\Delta^{-s}$, a term which is obviously of the $k$-th order of the eccentricities.

Then, neglecting powers and products of the eccentricities of the Earth and *Mars*, and

$$\{ r'''^3 + 2r''' r''^2 \} = (a''')^3 \{ (1 + 2\alpha^2 + 3e'''^2) - (3 + 2\alpha^2) e''' \cos \alpha''' + \ldots \quad - 4\alpha e'' \cos \alpha'' + \ldots \},$$

$$\{ r''^3 + 2r'' r'''^2 \} = (a''')^3 \{ (2 + \alpha^2 + 3e'''^2) \alpha - 2\alpha e''' \cos \alpha''' + \ldots \quad - (2 + 3\alpha^2) \alpha e'' \cos \alpha'' + \ldots \},$$

$$\cos (v - v''') = \cos (v - n'''t - \epsilon''') + \{ 2e''' \sin \alpha''' + \ldots \} \sin (v - n'''t - \epsilon'''),$$
$$\cos (v - v'') = \cos (v - n''t - \epsilon'') + \{ 2e'' \sin \alpha'' + \ldots \} \sin (v - n''t - \epsilon''),$$

$$\cos (v - 2v''' + v'') = \cos (v + [n'' - 2n'''] \, t + \epsilon'' - 2\epsilon''')$$
$$+ \{ 4e''' \sin \alpha''' - 2e'' \sin \alpha'' + \ldots \} \sin (v + [n'' - 2n'''] \, t + \epsilon'' - 2\epsilon'''),$$

$$\cos (v - 2v'' + v''') = \cos (v - [2n'' - n'''] \, t - 2\epsilon'' + \epsilon''')$$
$$+ \{ 4e'' \sin \alpha'' - 2e''' \sin \alpha''' + \ldots \} \sin (v - [2n'' - n'''] \, t + 2\epsilon'' - \epsilon'''),$$

where, for convenience, $\frac{a''}{a'''}$ has been denoted by $\alpha$, as usual, and $\alpha''$ and $\alpha'''$ denote the mean anomalies of the Earth and *Mars*.

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Further, if $\sigma$ be the ratio of the masses of the Sun and *Mars*, and $m = \dfrac{n''}{n}$,

$$m'' = \sigma m^2 \left(\frac{a'''}{a}\right)^3.$$

Substituting these values in the expression for $R^{(''')}$, it becomes

$$\begin{aligned} R^{(''')} = \sigma m^2 . \alpha^4 . \left(\frac{a}{a''}\right) \cdot \frac{r^3}{a^4} \cdot \Big\{ \frac{15}{16} \Big( & [1 + 2\alpha^2 + 3e'''^2] \, [B_{\frac{7}{2}}]^{24}_{21} - [2 + \alpha^2 + 3e'''^2] \, \alpha \, [B_{\frac{7}{2}}]^{23}_{20} \\ & + \alpha^2 \, [B_{\frac{7}{2}}]^{22}_{19} - \alpha \, [B_{\frac{7}{2}}]^{25}_{22} \\ & - \frac{1}{2} (1 - 2\alpha^2) \, e''' \, [B_{\frac{7}{2}}]^{24}_{22} + \alpha e''' \, [B_{\frac{7}{2}}]^{23}_{21} - \frac{3}{2} e''' \alpha^2 \, [B_{\frac{7}{2}}]^{22}_{20} - e''' \alpha \, [B_{\frac{7}{2}}]^{25}_{23} \\ & - 2\alpha e'' \, [B_{\frac{7}{2}}]^{23}_{21} - \frac{1}{2} (2 - \alpha^2) \, \alpha e'' \, [B_{\frac{7}{2}}]^{22}_{20} + \frac{3}{2} \alpha e'' \, [B_{\frac{7}{2}}]^{24}_{22} + \alpha e'' \, [B_{\frac{7}{2}}]^{21}_{19} \\ & + \frac{4}{5} [B_{\frac{5}{2}}]^{23}_{20} \alpha - \frac{4}{5} [B_{\frac{5}{2}}]^{24}_{21} \quad \Big) \cos \left( v - [24n'' - 20n'''] \, t - E \quad \right) + \ldots . \end{aligned}$$

The products of the eccentricities of *Mars* and the Earth, with the coefficients $[B_{\frac{5}{2}}]^{i}_{i+k}$, have been neglected because they will be very small.

In this expression, $r^3$ and $v$ can be directly replaced by $a''^3$ and $nt + \varepsilon$, as the terms depending on the eccentricity of the Moon, are not required. Then, putting the above coefficient in brackets equal to (R), we have

$$R^{(''')} = \sigma m^2 . \alpha^4 . \left(\frac{a}{a''}\right) . \frac{1}{a} . (R) \cos \{ [n - 24n'' + 20n'''] \, t + \epsilon - E \}.$$

Substituting this value in the equation for $\delta n$, since

$$\delta n = -3 \iint a n^2 \left(\frac{dR}{d\epsilon}\right) dt^2,$$

the inequality in the mean motion of the Moon will be

$$\delta n = -\sigma m^2 . \alpha^4 . \left(\frac{a}{a''}\right) . \left(\frac{n}{n - 24n'' + 20n'''}\right)^2 . 3 \, (R) . \sin \{ [n - 24n'' + 20n'''] \, t + \epsilon - E \}.$$

Adopting Hansen's value for the mean motion of the Moon (*Tables de la Lune*) and Leverrier's values for the mean motions of *Mars* and the Earth (*Annales de l'Observatoire*), then

$$\begin{array}{lcr} n & = & 17 \; 325 \; 594''\cdot 0 \\ 24n'' - 20n''' & = & 17 \; 322 \; 447''\cdot 5 \\ \hline n - 24n'' + 20n''' & = & 3 \; 146''\cdot 5 \end{array}$$

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The motion of the perihelions of *Mars* and the Earth may be neglected in what follows, though tending to slightly decrease the results.

Suppose $\sigma$, or the ratio of the masses of *Mars* and the Sun, to be taken as $\frac{1}{3{,}000{,}000}$, and assume $\frac{a}{a''} = 0{\cdot}0025856$, corresponding to a solar parallax of $8''{\cdot}85$; then

$$\sigma m^2 \alpha^4 . \left(\frac{a}{a''}\right) . \left(\frac{n}{n - 24n'' + 20n'''}\right)^2 . 3 = 12''{\cdot}5150 . \log = 1{\cdot}09753.$$

The coefficients $[B_{\frac{7}{2}}]^{i}_{i+k}$ can be readily calculated by making use of the developments given in either Pontécoulant's *Système du Monde*, or Delaunay's memoir in the *Conn. des Temps*, 1863.

On substituting the proper values in the above expression for (R), the first four terms entirely destroy each other, and with them vanishes the largest part of the coefficient of the term. The terms depending on the eccentricity of *Mars* yield therefore the principal part of the coefficient, those depending on the eccentricity of the Earth being decreased by its smallness. The last two terms destroy each other. The complete value is therefore

$$\begin{aligned}
&\overset{''}{+5{\cdot}90} \sin\{[n - 24n'' + 20n'''] t + \epsilon - 24\epsilon'' + 20\epsilon''' + 3\varpi'''\} \\
&-2{\cdot}33 \sin\{[n - 24n'' + 20n'''] t + \epsilon - 24\epsilon'' + 20\epsilon''' + \varpi'' + 2\varpi'''\} \\
&+0{\cdot}10 \sin\{[n - 24n'' + 20n'''] t + \epsilon - 24\epsilon'' + 20\epsilon''' + 2\varpi'' + \varpi'''\} \\
&+0{\cdot}04 \sin\{[n - 24n'' + 20n'''] t + \epsilon - 24\epsilon'' + 20\epsilon''' + 3\varpi''\}
\end{aligned}$$

or

$$\delta n = +7''{\cdot}55 \sin\{[n - 24n'' + 20n'''] t + \epsilon - 24\epsilon'' + 20\epsilon''' + 284^\circ\ 14'\}.$$

Writing it as a term increasing with the time, then

$$\delta n = \qquad \sin\{53'{\cdot}2\,(T - 1807{\cdot}6)\},$$

where T is the year for which the value of the inequality is required. The inequality has therefore a period of 406·0 years.

In the *Monthly Notices* for November 1873, there is a paper by Sir George Airy bearing on the lunar theory. The Astronomer Royal there remarks:—"It appears, therefore, that after applying the best correction that we are able to give to the assumed mean longitude and mean motion, there still remains a series of unexplained discordances, varying from $-6''{\cdot}79$ to $+4''{\cdot}93$. And each of the discordances here exhibited is based on about 440 observations; these observations are all reduced by the same system of clock stars and other elements (Bessel's in the *Tabulæ Regiomontanæ*), and the tabular places are all computed by the same theory (Plana's with Damoiseau's epochs).

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"I conceive it to be totally impossible that a complete and correct theory can leave such large and systematic discordances. And I express my opinion that there is still some serious defect in the lunar theory." (*Monthly Notices*, xxxiv., p. 5.)

The Astronomer Royal has tabulated these residual errors for groups of about $4\frac{1}{2}$ years, between 1750 and 1851. It was thus easy to see how far the newly discovered term reduces or augments these residual discordances. It was therefore gratifying to find that for the period between 1780 and 1851 it completely accounts for them. For this period the mean of the outstanding positive errors is reduced to about $0''\cdot7$, and the mean of the outstanding negative errors is reduced to about $0''\cdot6$. It does not account so well for the residual discordances between 1765 and 1780, and fails to account for the errors in the period 1750–1765. I have, however, discovered a minor term which will partially account for the errors in these earlier observations between 1750 and 1770.

From the nature of the new term it is obvious that the value which has been deduced for it in the present paper is at best merely provisional. It will require most careful revision, and the higher powers of the disturbing forces must be taken into account. This will involve some labour, because, owing to the great eccentricity of the orbit of *Mars*, the convergency of the series which it involves is slow.. It is probable that when the complete value of the term is ascertained, it will be found to be slightly smaller than that depending on the first power of the disturbing forces. Under these conditions it would still better represent the observations. The existence of this term is so strongly supported by observation, from the manner in which it removes the discordances between theory and observation detected by Sir G. Airy, that I thought it better to publish it in its present form, rather than defer this for the considerable period which may elapse before I am able to verify these results.

I take this opportunity to announce that I have discovered other terms in the mean motion of the Moon, arising from the direct action of the planets, and having sensible coefficients. I hope to be able to bring these before the Society in a short period. One of these, having the comparatively short period of 60 years, appears to be strongly indicated by observation.

*London, Nov.* 2, 1877.

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## *On Three Small Inequalities in the Mean Motion of the Earth, and an Inequality in the Mean Motion of Mars.* By E. Neison, Esq.

Whilst engaged on some investigations connected with the perturbations in the motion of the Moon produced by the action of the planets, I came across several terms of long period in the